Sample Questions I, Linear Algebra 2019

Sunday, April 28, 2019 11:01 AM

(1) which of the following is not true.

(a) A linear system is inconsistent if and only if an echelon form of the augmented matrix has a row of the form $[0\ 0 \cdots 0\ b]$ where $b \neq 0$.

(b) If in the augmented matrix, the number of pivot positions is the same as the number of columns, then the system is consistent.

(c) If an echelon form of the augmented matrix does not have a row of the form $[0\ 0 \cdots 0\ b]$ where $b \neq 0$, and the number of pivot positions is equal to the number of columns minus 1, then the linear system has only one solutions.

(d) If the system is consistent and has a free variable, then, the system has infinitely many solutions.

(2) Determine which matrices are in reduced echelon form and which others are only in echelon form.

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(3) Determine the value(s) of h such that the following matrix the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix}$$

(a) $\mathbb{R} \setminus \{15\}$ (b) $\mathbb{R} \setminus \{-15\}$ (c) 15 (d) -15

Sample Questions II, Linear Algebra 2019

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(4) Mark each statement true or false.

(a) The echelon form of a matrix is unique

(b) The reduced echelon form of a matrix is unique.

(c) whenever a system has a free variable it has infinitely many solutions.

(d) whenever all variables of a linear system are basic, the system has only one solution.

(5) A set of vectors $\{v_1, \dots, v_p\}$ spans $\mathbb{R}^n$ if

(a) every vector $v \in \mathbb{R}^n$ can be written as the form of $v = c_1 v_1 + c_2 v_2 + \dots + c_p v_p$ where $c_1, \dots, c_p \in \mathbb{R}$.

(b) every vector in $\mathbb{R}^n$ can be uniquely written as a linear combination of the vectors $v_1, \dots, v_p$.

(c) If $c_1 v_1 + c_2 v_2 + \dots + c_p v_p = 0$ for some $c_1, c_2, \dots, c_p$ in $\mathbb{R}$, then $c_1 = c_2 = \dots = c_p = 0$.

(d) For every $v \in \mathbb{R}^n$, the system $c_1 v_1 + \dots + c_p v_p = v$ has only one solution.

(6) A set of vectors $\{v_1, \dots, v_p\}$ in $\mathbb{R}^n$ are linearly independent i

(a) every vector $v \in \mathbb{R}^n$ can be written as the form of $v = c_1 v_1 + c_2 v_2 + \dots + c_p v_p$ where $c_1, \dots, c_p \in \mathbb{R}$.

(b) every vector in $\mathbb{R}^n$ can be uniquely written as a linear combination of the vectors $v_1, \dots, v_p$.

(c) If $c_1 v_1 + c_2 v_2 + \dots + c_p v_p = 0$ for some $c_1, c_2, \dots, c_p$ in $\mathbb{R}$, then $c_1 = c_2 = \dots = c_p = 0$.

(d) For every $v \in \mathbb{R}^n$, the system $c_1 v_1 + \dots + c_p v_p = v$ has only one solution.

Sample Questions III, Linear Algebra 2019

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(7) A set of vectors $\{v_1, \ldots, v_p\}$ is $\mathbb{R}^n$ are linearly dependent if the vector equation $c_1v_1 + \cdots + c_pv_p = 0$ has

(a) a unique solution.

(b) at least one solution.

(c) at least one non-zero solution.

(d) only one nonzero solution.

(8) A set $S = \{v_1, \ldots, v_p\}$ of two or more vectors in $\mathbb{R}^n$ linearly independent if

(a) at least one of the vectors in S is a linear combination of the others.

(b) $p < n$.

(c) $p > n$.

(d) the matrix $[v_1\ v_2\ \cdots\ v_p]$ has p pivot positions.

(9) Let be $\mathbb{R}^3$. which of the following matrix equations has infinitely many solutions (I is the identity matrix, and 0 is the zero matrix).

(a) $IX = b$ (b) $0X = b$ (c) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} X = b$ (d) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} X = b$

(10) Mark each statement True or False.

(a) A homogeneous equation is always consistent.

(b) The homogeneous equation $AX = 0$ has the trivial solution if and only if the equation has at least one free variable.

(c) The equation $AX = b$ is homogeneous if the zero vector is a solution.

(d) If x is a nontrivial solution of $AX = 0$, then every entry in x is nonzero.

(e) Either three cars or a SUV are in the front of the Department of Mathematics.

(f) Either a SUV or three cars are in the front of the Department of Mathematics.

Sample Questions IV, Linear Algebra 2019

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(11) Mark each statement True or False.

(a) If x and y are linearly independent, and if $\{x, y, z\}$ is linearly dependent, then z is in Span$\{x, y\}$.

(b) If x and y are linearly independent, and if z is in Span$\{x, y\}$, then $\{x, y, z\}$ is linearly dependent.

(c) If a set in $\mathbb{R}^n$ is linearly dependent, then the set contains more vectors than there are entries in each vector.

(d) The columns of any 4×5 matrix are linearly dependent.

(12) If $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a linear transformation, then

(a) $T(0) = 0$

(b) $T(cx + dy) = cT(x) + dT(y)$ for all $x, y \in \mathbb{R}^n$ and $c, d \in \mathbb{R}$.

(c) $T(cx - dy) = cT(x) - dT(y)$ for all $x, y \in \mathbb{R}^n$ and $c, d \in \mathbb{R}$.

(d) All (a), (b), and (c) are true statements.

(13) Let $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear transformation with the standard matrix A. Let $n \geq m$.

(a) If A has pivot position in each row, T is one-to-one.

(b) If A has pivot position in each column, T is onto.

(c) If A has pivot positions in all rows and columns, T is an isomorphism.

(d) T is always onto.

(14) Mark each statement True or False.

(a) Every linear transformation has a standard matrix.

(b) If $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a linear transformation and $m > n$ T is one-to-one.

(c) If $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a linear transformation and $m > n$ T is onto.

(d) If $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is onto, then we must have $n \geq m$.

Sample Questions V, Linear Algebra 2019

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(15) If A, B, and C are $n \times n$ matrices. Then

(a) we have $AB = BA$.

(b) If $AB = AC$, then $B = C$.

(c) If $AB = 0$, then A or B is zero.

(d) $(AB)^T = B^T A^T$.

(16) Let A be an $n \times n$ matrix, and $b \in \mathbb{R}^n$.

(a) If A is invertible, then $Ax = b$ has only one solution, that is $x = A^{-1}b$.

(b) If A is not invertible, then the columns of A are linearly independent.

(c) If A is not invertible, $Ax = b$ has atleast one solution.

(d) If A is invertible, it is possible that $Ax = b$ has infinitely many solutions.

(17) Let $A \in M_{n \times n}(\mathbb{R})$. which one is not equivalent to the others.

(a) There is an $n \times n$ matrix C such that $AC = I$.

(b) The columns of A are linearly dependent.

(c) The linear transformation $x \mapsto Ax$ is not one-to-one.

(d) the equation $Ax = 0$ has infinitely many solutions.

18) Determine which of the following matrices are invertible.

(a) $\begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 2 \\ 1 & 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 7 \\ -3 & -6 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & -1 & \\ 0 & 0 & 0 & 4 \end{bmatrix}$

(19) If $A, B \in M_{n \times n}(\mathbb{R})$, and AB is invertible, then

(a) A is invertible (b) B is invertible (c) A and B are invertible.

(d) A and B are not invertible.

Sample Questions VI, Linear Algebra 2019

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(20) If A is an $n \times n$ matrix. Then

(a) The null space of A is not a subspace of $\mathbb{R}^n$.

(b) The null space of A is the solution set of $Ax=0$.

(c) The dimension of null space of A is equal to the number of pivot positions.

(d) The pivot columns form a basis for null space of A.

(21) If A is an $n \times n$ matrix. Then

(a) $\text{rank } A + \dim \text{Nul } A = n$.

(b) $\text{rank } A + \dim \text{Nul } A > n$.

(c) $\text{rank } A + \dim \text{Nul } A < n$.

(d) $\text{rank } A + \dim \text{Nul } A = 0$.

(22) Let H be a subspace of dimension p of $\mathbb{R}^n$. which of the following is not true.

(a) The maximum number of linearly independent vectors in H is p.

(b) If p vectors spans H, then they are linearly dependent.

(c) Any linearly independent set of p vectors in H spans H.

(d) If a set of $p+1$ vectors spans H, then that set is not linearly independent.

(23) Mark each statement True or False (A is in $M_{n \times n}(\mathbb{R})$).

(a) If the columns of A span $\mathbb{R}^n$, then the columns of A are linearly independent.

(b) If the columns of A are linearly independent, then the columns of A span $\mathbb{R}^n$.

(c) If A^T is not invertible, then A is not invertible.

(d) If the equation $Ax=b$ has at least one solution for each $b \in \mathbb{R}^n$, then the solution is unique for each b.

Sample Questions VII, Linear Algebra 2019

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(24) If $A = \begin{bmatrix} a & b & e \\ d & e & f \\ g & h & i \end{bmatrix}$ with $\det A = -3$, then

$$\det \begin{bmatrix} -3a & -3d & -3g \\ 3b & 3e & 3h \\ 3c & 3f & 3i \end{bmatrix} =$$

(a) 3^4 (b) -3^4 (c) 3^2 (d) -3^2

(25) Mark each statement True or False.

(a) A row replacement operation does not affect the determinent of a matrix.

(b) If the columns of A are linearly independent, then $\det A = 0$.

(c) $\det A^{-1} = (-1) \det A$.

(d) $\det (AB)^{-1} = \det(B^{-1}A^{-1}) = \det(B^{-1}) \det(A^{-1})$.